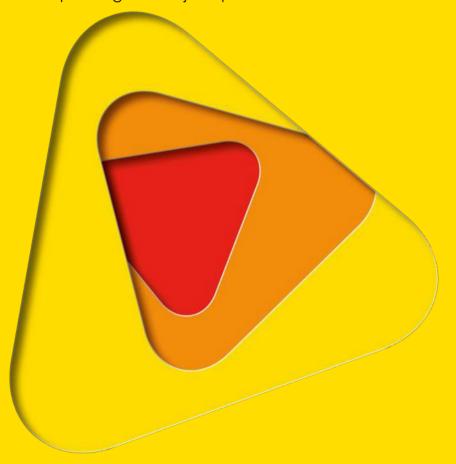


THRESHOLD BEHAVIOUR IN DYNAMIC FIRE PROPAGATION

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ABSTRACT

Recent research has demonstrated that under conditions of extreme fire weather, bushfires burning in rugged terrain can exhibit distinctly dynamic patterns of propagation, which can have a dramatic effect on subsequent fire development. Coupled fire-atmosphere modelling using large eddy simulation has been useful in shedding light on the physical mechanisms underlying these phenomena, for example highlighting the important role of fire-induced vorticity. In particular it has confirmed that the onset of dynamic modes of fire propagation is subject to a number of environmental thresholds. This is not the first time that the existence of threshold behaviour in combustion-related systems has been identified. In this paper we provide a brief summary of some combustion-related systems that exhibit threshold behaviour. Specifically we discuss the emergence of dynamic modes of fire propagation in exceedingly simple representations of combustion systems, and the existence of environmental thresholds relating to the propagation of wildfires in rugged terrain. Most significantly, we present new research that specifically investigates the environmental precursors necessary to drive a particular type of dynamic fire propagation known as vorticity-driven lateral spread (VLS). This research extends previous coupled fire-atmosphere modelling, to specifically consider the effect of wind speed and topographic slope ingenerating the fire-induced vorticity necessary to drive VLS.

The modelling results indicate the existence of environmental thresholds beyond which VLS is likely to occur. The results also indicate that the transition from quasi-steady to dynamic fire propagation can be quite abrupt, requiring only minimal changes in wind speed and slope for onset. The propensity for dynamic interactions to produce erratic and dangerous fire behaviour has strong implications for firefighter and community safety.

Keywords: Dynamic fire spread, VLS, threshold, bifurcation, extreme fire behaviour, wind-terrain-fire interaction.

INTRODUCTION

Dynamic escalation of wildland fires into large conflagrations represents a significant challenge to the management of fires in the landscape. Multi-scale interactions between a fire and the local environment, which includes fuels, weather and topography, can produce highly complex patterns of fire spread that are currently beyond the capabilities of operational fire spread models. Understanding the physical processes that underpin these complex modes of fire propagation is a key step in developing the next generation of fire propagation models and in improving the way extreme bushfires are managed. Recent research into the behaviour of wild-fires has identified a number of dynamic modes of fire propagation. These modes of fire spread are referred to as dynamic because they are manifestly at odds with quasi-steady fire propagation, whereby a fire spreads at an approximately constant rate given uniform environmental conditions.

Viegas (2005) and Dold and Zinoviev (2009) examined the ability of a fire to exhibit exponentially increasing rates of spread up steep slopes and canyons, while Viegas et al. (2012) discussed the abrupt increases in rate of spread that can occur when two lines of fires intersect at some oblique angle. Another form of dynamic fire propagation was identified by Sharples et al. (2012) in connection with the 2003 Canberra bushfires. This phenomenon, which will be referred to as vorticity-driven lateral spread (VLS), involves the rapid lateral propagation of a fire across a lee-facing slope in a direction approximately perpendicular to the prevailing wind direction.

While the emergence of dynamic modes of fire propagation in wildfire situations has only recently begun to attract particular attention from researchers, the existence of dynamic modes of fire propagation in other combustion-related systems has long been acknowledged and studied (Zel'dovich et al. 1985; Weber et al. 1997; Merzhanov and Rumanov 1999; Gubernov et al. 2003; Sharples et al. 2009). Indeed, it has been shown that even the most elementary mathematical representations of combustion systems allow for the possibility of dynamic modes of fire spread (e.g. Sharples et al. 2009). These arise as a consequence of the inherent properties of dynamical systems and are defined by bifurcation (i.e. threshold) points relating to the various system parameters.

Although these simple combustion systems differ quite markedly from those encountered in wildfire situations, it is interesting to note the presence of dynamic modes of propagation in even the most simple representations of combustion systems. Given the inherent dynamics of simple combustions systems, it is then not surprising that dynamic modes of fire propagation also exist in wildfire situations.

In this paper we provide a brief summary of the various forms of dynamic fire propagation that arise in a number of situations. We begin with a very simple representation of a combustion system, namely that of a combustion wave propagating through a fuel bed. A mathematical model for the propagation of the combustion wave is derived based on single-step chemistry with Arrhenius kinetics and conservation of energy and mass. Despite the simplistic nature of the model system, dynamic modes of fire propagation are a mathematical certainty (Gubernov et al. 2003; Sharples et al. 2009) and we present some examples of these dynamic propagation modes. We next consider the phenomenon of flame attachment and discuss how it is critically dependent on the slope of the terrain. The vertical rise of flames into the air, which is perhaps considered typical, will in fact only occur on slopes below a threshold angle. For fires burning on slopes inclined above this threshold angle, dynamic interactions between the fire, the atmosphere and the terrain can cause the fire's plume to attach to the surface. In these instances the surface will be effectively bathed in flames. While this effect is well understood for certain geometric configurations, there is little knowledge of how the effect manifests in terrain geometries that are found in the landscape. We report on some initial progress in this area and discuss plans for future work. Finally, we turn our attention to the VLS phenomenon and provide a brief account of some recent findings relating to environmental thresholds for VLS occurrence. In particular we consider the sensitivity of VLS to changes in wind and topographic slope.

COMBUSTION WAVE DYNAMICS

To illustrate the emergence of dynamic modes of fire propagation in simple combustion systems, we will consider the propagation of a combustion wave through a reactive medium. Figure 1a gives an example of such a phenomenon. The fuel is ignited at the top and the fire spreads down through the fuel as a front, which can be thought of as the interface between burnt and unburnt fuel. Such a front is an example of a combustion wave. To determine the profile and speed of a combustion wave, we invoke conservation of energy and conservation of mass (i.e. fuel). Applying the appropriate book-keeping to the system, it follows that the combustion wave at time t and position x is given by the (non-dimensional) temperature u(x, t) and the fuel-fraction v(x, t), which are determined through solution of the system of partial differential equations (Weber et al. 1997):

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + ve^{-1/u} \tag{1}$$

$$\frac{\partial v}{\partial t} = Le^{-1}\frac{\partial^2 v}{\partial x^2} - \beta v e^{-1/u} \tag{2}$$

Note that equation (1) guarantees conservation of energy, while equation (2) guarantees conservation of fuel. The system parameters Le and β , describe the state of the fuel, and the exothermicity of the combustion reaction, respectively. Note that smaller values of β correspond to more exothermic reactions.

Figure 2 shows how the speed of the combustion wave varies with time when Le=8 and β assumes various values. For instance, when Le=8 and $\beta=7.5$ we find that the combustion wave propagates with a constant speed of just over 0.02 (Figure 2a). However, if the value of β is increased from $\beta=7.5$ to $\beta=7.6$, we find that now the combustion wave propagates with a wave speed that varies periodically; that is, the fire will spread with a speed that continually oscillates between 0.01 and 0.035, approximately (Figure 2b). Figures 2c and 2d, show that as β is increased further, more complicated dynamic combustion wave propagation arises. Figure 2c shows a doubly-periodic wave speed, while Figure 2d shows an essentially chaotic wave speed.

The dynamic modes of fire propagation evident in Figure 2b arise due to the presence of a Hopf bifurcation in the dynamical system representing the combustion reaction. The more complicated behaviours seen in Figures 2c and 2d arise through the presence of further period-doubling bifurcations. Mathematical niceties

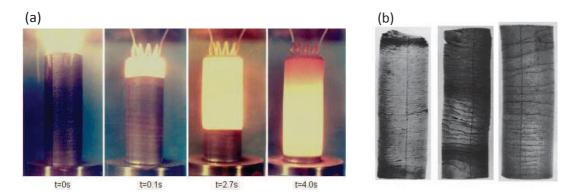


Figure 1. (a) A combustion wave propagates through a reactive medium in an example of the Self-propagating High-temperature Synthesis (SHS) process (b) laminar defects in SHS products due to dynamic (periodic) variations in combustion wave speed. The figure has been adapted from Makino (2000).

aside, the main point is that even in the most elementary representations of combustion systems, there are inherent dynamics that can emerge under certain configurations of the system. Moreover, the emergence of dynamic behaviour in these simple combustion systems has real and important consequences. Figure 1b shows the consequences of a dynamic wave speed in an industrial combustion process called self-propagating high-temperature synthesis (Makino 2000).

We now extend our attention to consider the emergence of dynamic behaviour of fires burning in steep and confined terrain.

TERRAIN-RELATED THRESHOLDS FOR FLAME ATTACHMENT

It appears the Rothermel (1984) was the first to discuss the possibility of flames attaching to a sufficiently inclined surface. Indeed, Rothermel (1984) specifically states that:

There is no definitive research on the problem of flame attachment. It appears from both lab work and discussions with users that the flame becomes attached near 50 percent slope with no prevailing wind. It is not known, but it may not be possible to produce a fire of enough intensity to cause flame detachment on steep slopes regardless of the method of ignition.

In this context, it is of interest to note that a 50 percent slope is equivalent to a slope of 26.6°.

The first scientific investigations into the flame attachment phenomenon were conducted as part of the official investigation into the cause of the disastrous development of an escalator fire in the London Underground (Crossland 1992; Drysdale et al. 1992; Moodie and Jagger 1992; Smith 1992; Wu and Drysdale 1996). A fire that started on an escalator, just below the ticket hall at King's Cross Station, was initially assessed by firefighters as being of only minor concern. However, in an unexpectedly short amount of time the fire spread with extreme ferocity up the escalator trench and into the ticket hall and surrounding areas with tragic consequences. The fire ultimately killed 31 people – it was described by witnesses as a *blowtorch* or an *eruption*. It is interesting to note that the King's Cross Fire occurred in 1987, approximately 3 years after Rothermel's publication in 1984, and that remarkably, the investigation into the King's Cross Fire found that flames would attach to the surface of the escalator trench when it was inclined over about 26° (Simcox et al. 1992; Drysdale and Macmillan 1992; Wu et al. 2000).

The mechanism that drives flame attachment is a dynamic transition from separated to attached flow of the fire's plume. The interaction between the buoyant plume and the atmosphere creates indrafts into the base of the fire. However, when the terrain surface is inclined above a critical, or threshold angle of about 26°, the entrainment into the plume from below the fire cannot be matched by the entrainment from above the fire. This results in a pressure deficit near the surface directly upslope from the location of the fire. At this point the flow associated with the fire's plume encounters a bifurcation, which results in the plume attaching to the surface. This in turn causes enhanced preheating of the fuels above the fire, as they are

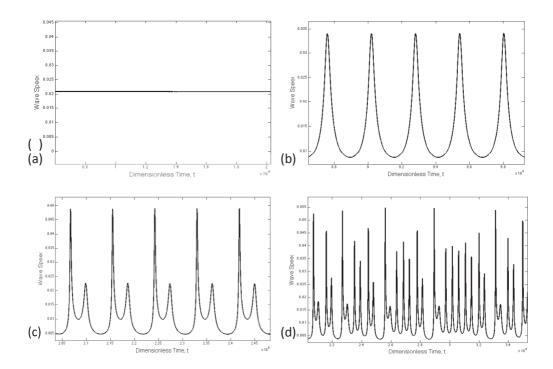


Figure 2. Plots of combustion wave speed versus time (Le=8). (a) Constant speed propagation when $\beta=7.5$, (b) periodic combustion wave speed when $\beta=7.6$, (c) period-2 variation in wave speed when $\beta=7.9$, (d) chaotic wave speed when $\beta=7.95$.

bathed in the hot plume of the fire moving along the surface. This enhanced preheating then results in enhanced rates of spread. Indeed, Dold and Zinoviev (2009) indicate that there is no obvious upper limit to the speed at which an attached flame can move over a surface¹.

Research into flame attachment has focused on two cases: that of an inclined planar surface; and that of an inclined rectangular trench. While such cases are of interest, particularly in a structural firefighting context, they do not tend to correlate well with the elements of terrain encountered in the landscape. For example, canyons or gullies within a landscape tend to be somewhat V-shaped in profile, rather than rectangular like an escalator trench. Moreover, due to the effects of erosion, most gullies or canyons have a layer of sediment along their waterline, which tends to smooth the V-shaped profile.

Experimental fires set in steep V-shaped canyons indicated that the threshold angle of inclination for such a configuration is in the range of 30° – 40° . Examples of the observed fire behaviour can be seen in Figure 3. In Figure 3a the canyon is inclined at 30° and the fire spreads up the canyon as two distinct head fires that follow the lines of maximum slope. Figure 3b shows clearly that for this case the fire's plume is not attached to the terrain surface. Figure 3c shows that when the canyon is inclined at 40° the resultant pattern of fire spread is markedly different to the 30° inclination case. In Figure 3c the fire can be seen propagating as a single head fire straight up the waterline of the canyon. Moreover, Figure 3d shows that the fire's plume is now attached to the surface – the flames are only visible from the side view as they erupt from the top of the canyon.

The flame attachment phenomenon therefore serves as another example of dynamic fire propagation, which is subject to threshold behaviour. In this instance the threshold relates to the conditions of the terrain. Investigations to date indicate that the threshold angle of inclination, above which flames will attach to a surface, is dependent upon the geometric profile of the canyon. This is being investigated in further research funded through the Australian Research Council.

¹ Of course, one would not expect flames to exceed the speed of sound!

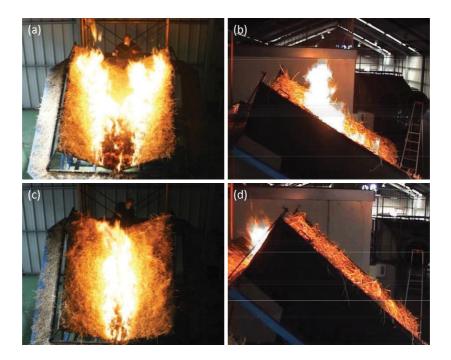


Figure 3. Photographs of experimental fire behaviour in V-shaped canyons. In panels (a) and (b) the canyon is inclined at 30°, while in panels (c) and (d) the canyon is inclined at 40°.

THRESHOLDS TO VORTICITY-DRIVEN LATERAL SPREAD

Vorticity-driven lateral spread (VLS) is a dynamic mode of wildfire propagation that has been linked to the rapid development of bushfires (Sharples et al. 2012; Simpson et al. 2013; Simpson et al. 2014). Initial work aimed at understanding the processes underlying the VLS phenomenon identified topographic slope and aspect conditions that were conducive to VLS occurrence. Specifically, it was found that a necessary condition for VLS occurrence was given by $\chi(\sigma, \delta) = 1$, where

$$\chi(\sigma, \delta) = \begin{cases} 1, & \text{if } \gamma_s \ge \sigma \text{ and } |\theta_w - \gamma_a| \le \delta, \\ 0, & \text{otherwise.} \end{cases}$$
 (3)

In equation (3), γ_s and γ_a are the topographic slope and aspect angles, respectively, and θ_w is the wind direction angle (i.e. the way the wind is heading). The model (3) is defined by the model parameters σ and δ , which are termed the *slope threshold* and *aspect discrepancy threshold*, respectively. Sharples et al. (2012) found that for VLS events observed during the 2003 Canberra fires, the model parameters could be estimated as $\sigma \approx 25^\circ$ and $\delta \approx 40^\circ$. This model has also been successfully applied to several other notable fires, though these studies are yet to be published.

Sharples et al. (2013) examined the effect of variable prevailing wind speed on VLS occurrence. They considered an idealised triangular ridge with an apex line oriented orthogonally to the prevailing wind directions. Further details of the simulations are provided by Simpson et al. (2013) and Sharples et al. (2013). For wind speeds below about 5 ms⁻¹ it was found that VLS did not occur. The reason for this is that under such low wind speeds the flow does not possess enough momentum to separate from the surface in the lee of the ridge line. As a consequence, the is no source of ambient vorticity and the fire cannot produce the interaction that drives VLS. On the other hand, for wind speeds of 7.5 ms⁻¹ or greater the simulations all exhibited VLS (see Figure 4).

Simpson et al. (submitted) extended the study described above, by systematically altering the inclination of the leeward slope while holding the prevailing wind speed at a constant 15 ms⁻¹. They found that for slopes below 20° the spread of a simulated fire ignited on the leeward slope was dominated by spread in a downslope direction. This again reflects the nature of the wind flow driving the fire – in these low slope



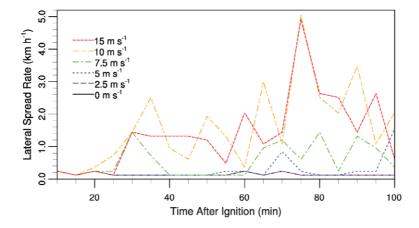


Figure 4. Lateral spread rates arising from coupled fire-atmosphere simulations of fires burning on leeward slopes under different prevailing wind speeds.

cases, the lee slope is not sufficiently steep to drive the flow separation required to create the ambient horizontal vorticity required for VLS. Instead the wind simply flows over the ridge line and drives the fire as a downslope wind. For leeward slopes of 25° the pattern of fire spread is indicative of a transitional stage between the quasi-steady behaviour in the shallower slope cases and the dynamic behaviour for steeper leeward slopes. Indeed, for leeward slopes of 25° or more the pattern of fire spread is markedly different, with the evolution of the fire's perimeter dominated by rapid lateral spread across the top of the leeward slopes. This transition to dynamic fire propagation as the leeward slope angle is increased is illustrated in Figure 5.

Ongoing research is examining environmental thresholds relating to topographic aspect and properties of the fuel, as well as the interdependence of these thresholds as environmental factors are varied in combination.

DISCUSSION AND CONCLUSIONS

Thresholds to dynamic behaviour are inherent in combustion systems, even in their most elementary realisations. Dynamic instabilities in combustion systems can give rise to variable propagation speed (e.g. oscillatory) that result in undesirable consequences for industrial processes (e.g. impurities, thermal runaway). We have demonstrated that dynamic fire behaviour can also arise in wildfire situations, but that its occurrence can depend critically on a number of environmental factors; for example, in V-shaped canyons there is a slope threshold above which flames will attach to the surface. Flame attachment can result in significantly enhanced (and accelerating) rates of fire spread and may have played a role in a number of wildfire incidents in which firefighters were injured or killed (Sharples et al. 2010).

Investigation of flame attachment in the types of terrain geometries found in the landscape is still in its initial stages. A number of challenges remain. For example: How does the threshold inclination for flame attachment change in response to variations in the geometric profile of the canyon?; and How does the presence of an ambient wind affect the threshold inclination? These issues will be addressed in ongoing research.

Most significantly, at least in the context of extreme wildfire development, is the VLS phenomenon and its associated environmental thresholds. The ability of wildfires to exhibit rapid lateral spread across steep, leefacing slopes, poses a significant challenge to the management of fires in rugged terrain. The interaction between the terrain-modified winds and the fire's plume transforms the ambient horizontal vorticity into a pair of counter-rotating vertical vortices, or fire-whirls, which drive the fire laterally across the slope. As demonstrated above, VLS is subject to a number of environmental thresholds, including those relating to wind speed, topographic slope and aspect and fuel properties.

The wind speed threshold for VLS, as determined using couple fire-atmosphere simulations, is approximately 5 ms^{-1} (18 km h^{-1}), which is in general agreement with the value of 20 km h^{-1} suggested by Sharples et al. (2012). This threshold relates to the nature of the wind flow over the leeward slope - the wind must

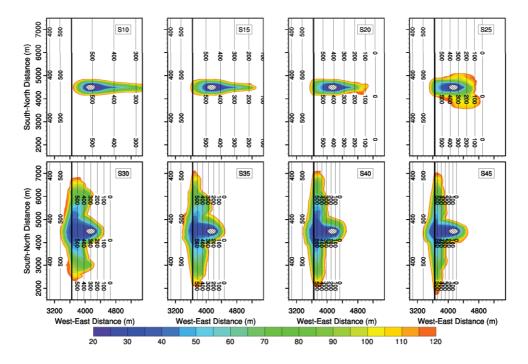


Figure 5. Coupled fire-atmosphere simulations of fires burning on leeward slopes with different inclinations. Slope values are indicated in the figure panels, for example 'S35' indicates a leeward slope angle of 35°.

be sufficiently strong to separate from the surface and produce horizontal vorticity, which is a key element of the VLS mechanism. The coupled simulations also indicated a topographic slope threshold for VLS of approximately 25°, which is in good agreement with the slope threshold determined empirically by Sharples et al. (2012). Again this threshold relates to the nature of the flow on the leeward slope. For leeward slopes inclined at an angle less than 25° the wind manifests essentially as a (non-separated) laminar flow and drives the fire downslope. On leeward slopes greater than 25° the leeward wind regime is dominated by turbulent eddies (which manifest as an upslope time-averaged flow). In such cases horizontal vorticity is present to drive the VLS phenomenon.

We have also considered thresholds relating to topographic aspect and fuel properties. Although this work is still under formal peer-review, we can report that the coupled simulations indicated a topographic aspect discrepancy threshold of approximately $10 - 20^{\circ}$. This means that the wind direction and topographic aspect direction must align to within $10 - 20^{\circ}$ of each other for VLS to occur. This is much less than the value reported by Sharples et al. (2012) of 40° . The reasons for this difference are not presently understood, though our suspicion is that it is due to the variable nature of environmental conditions in real situations, as opposed to the precise conditions that are set in the coupled simulations. VLS has also been found to be sensitive to fuel properties, as defined in the fuel classification framework of Anderson (1982). The most important fuel-related variables determining VLS occurrence appear to be wind speed reduction factor, initial fuel mass loading, mass loss rate and packing ratio. Essentially all of these variables affect the heat output from the fire. If the heat output is insufficient then VLS will not occur (Simpson et al. 2013).

The existence of environmental thresholds to dynamic fire propagation has a number of operational and safety implications. Indeed, they imply that small changes in environmental conditions can result in significant changes in the nature of fire spread, not to mention the associated fire behaviour characteristics. For example, with a slight change in topographic aspect, firefighters working on leeward slopes could suddenly encounter an abrupt and distinct change in fire behaviour: Figure 6 shows exactly this situation.

The fire near point A is propagating in a typical fashion, as is evident by the relatively thin flaming zone (yellow) around the fire perimeter. Similarly the fire at point B is progressing downslope towards the west (and the fire at point A) in a typical fashion. This fire behaviour has all the hallmarks of a well-executed backburn, ignited to head-off the main fire at point A. Note that a fire trail runs along the ridge line evident in

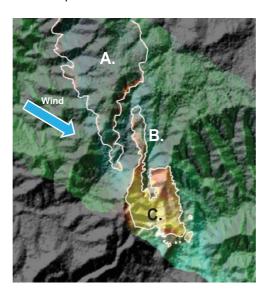


Figure 6. Multispectral linescan of a wildfire (point A) and a back-burn (points B and C).

the figure. The fire at point C exhibits a markedly different and far more alarming pattern of spread. The fire at this point appears to be in a state of *deep flaming*, where a large area of the landscape has been ignited in rapid succession. The fire at point C now dominates the fire propagation, and the associated pyroconvection, in what amounts to a phenomenological coup d'etat.

In fact the fire behaviour at point C is consistent with the VLS phenomenon, and demonstrates the topographic aspect threshold effect. The terrain at point B has an approximately northeasterly aspect that does not sufficiently align with the wind direction (wind blowing towards the southeast) to drive any dynamic fire spread. However, upwind of point C (and downwind of the fire trail along the ridge line) the topographic aspect is approximately southeasterly and aligns with the wind in a way that is highly conducive to VLS occurrence. The apparent lateral spread, the downwind extension of the flaming zone and the abundance of spot fires evident in the linescan near point C are all consistent with VLS. Indeed, it is difficult to account for the significant differences in fire behaviour, under meteorological and vegetation conditions that were more or less uniform over the region, without invoking some form of dynamic threshold behaviour.

Some may view the exacerbation of the fire near point C in Figure 6 as a failure of the fire crews involved or the strategic decisions that were made surrounding the ignition of the back-burn. However, in our opinion, the real failure was in the inadequacy of our collective understanding of how fires burn under extreme conditions in rugged terrain. Bushfires burning in rugged terrain under extreme weather conditions are distinctly dynamic phenomena that are influenced by a range of mechanisms that we are only beginning to properly understand. Incorporating new findings into training programs and increasing the general awareness of processes such as VLS, which have been shown to dramatically influence fire development, appear to be the best options for improving matters, at least in the immediate future.

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